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TECHNICAL NOTES

Oscillatory mixed convection in horizontal porous layers locally heated from below

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INTRODUCTION

IN RECENT years, mixed convection in porous media has received increased attention because of its important applications in geophysics and energy-related engineering problems. Indeed, in many of these applications, a situation frequently encountered is the existence of an externally induced flow. No matter how small it might be, superimposition of this forced flow needs consideration, since it can affect the flow structure and the total heat transfer significantly.

Recently, extensive numerical results for steady-state mixed convection in horizontal porous layers have been presented by the authors [1, 2]. The interaction between the buoyancy effects and forced flow is found to be very complicated. Introducing a forced flow does not always enhance the heat transfer rate. It has been reported that, for $Ra \geqslant 100$, there exists a 'critical' Peclet number for which the total heat transfer is a minimum [1]. It has also been pointed out by the authors [2] that steady-state analysis does not always lead to a converged solution for the case when the horizontal extent of the heat source is greater than three times the layer depth for $Ra \geqslant 100$ and $Pe \geqslant 2$. Instead, an oscillation in the temperature and flow fields is observed. In order to investigate this phenomenon further, a transient analysis is required. Therefore, the purposes of the present study are to perform a numerical, transient analysis and to investigate the origin of instability and the process of the flow transition.

FORMULATION AND NUMERICAL METHOD

The geometry is a two-dimensional layer bounded by two horizontal impermeable walls through which a flow of uniform velocity and constant temperature is induced (Fig. 1). The top wall is kept at a constant temperature while the bottom wall other than the heat source is adiabatic.

Having invoked the Boussinesq approximation, the dimensionless governing equations based on Darcy's law are given by [3]

$$\frac{\partial^2\psi}{\partial X^2}+\frac{\partial^2\psi}{\partial Y^2} = -\frac{Ra}{Pe}\frac{\partial\theta}{\partial X} \quad (1)$$

$$\frac{\partial\theta}{\partial\tau} = \left[\frac{\partial^2\theta}{\partial X^2}+\frac{\partial^2\theta}{\partial Y^2}\right] - Pe\left[\frac{\partial\psi}{\partial Y}\frac{\partial\theta}{\partial X}-\frac{\partial\psi}{\partial X}\frac{\partial\theta}{\partial Y}\right] \quad (2)$$

with the corresponding initial and boundary conditions

$$\theta(X, Y, 0) = 0 \quad (3)$$

$$\psi(X, Y, 0) = Y \quad (4)$$

$$\psi(X, 1, \tau) = 1, \quad \theta(X, 1, \tau) = 0 \quad \text{on the top of the surface} \quad (5)$$

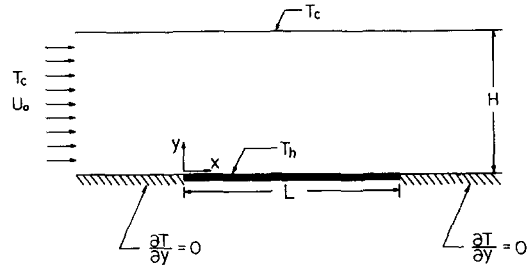

FIG. 1. A two-dimensional horizontal porous layer with a finite heat source.

$$\psi(X, 0, \tau) = 0, \quad \theta(X, 0, \tau) = 1 \quad \text{on the heat source} \quad (6a)$$

$$\frac{\partial\theta}{\partial Y}(X, 0, \tau) = 0 \quad \text{on the bottom surface, other than the heat source} \quad (6b)$$

$$\frac{\partial\psi}{\partial X}(X, Y, \tau) = 0, \quad \frac{\partial\theta}{\partial X}(X, Y, \tau) = 0 \quad \text{for } X \gg 0. \quad (7)$$

The dimensionless governing equations, equations (1) and (2), are discretized using the control volume approach. The convective terms in the energy equation have been approximated by the upwind scheme and the time derivative term by the forward differences. The stability constraint of the successive-substitutive formulation thus obtained is primarily on the time step as discussed by Jaluria and Torrance [4]. For convenience, uniform grids, 376×26, have been chosen for the present study. As reported in the previous study [1], the choice of this grid arrangement gives a very satisfactory result. The time step has been carefully chosen ($\Delta\tau = 10^{-4}$) such that the stability is guaranteed and accuracy is ensured. It is assumed that a steady state is reached when the changes in the stream function and temperature, as well as the local heat flux on the top and bottom walls, are less than 10^{-4} in consecutive time steps.

RESULTS AND DISCUSSION

The focus of the present study will be on the origin of the oscillatory convection in a porous layer for the special case of $A = 3$ since it has been reported that the flow and temperature fields exhibit an oscillating variation in a regime of $Ra \geqslant 100$ and $Pe \geqslant 2$ [3]. For other cases, detailed discussion can be found in ref. [3].

To reveal the complexity of the interaction between the forced flow and buoyancy effects, the temporal variations of the flow field are shown in Figs. 2 and 3 for $Pe = 0.5$ and 5,

NOMENCLATURE

- A ratio of the length of the heat source to the height of the porous layer, L/H
- A_p amplitude of oscillation
- c specific heat of fluid at constant pressure [$\mathrm{J\,kg^{-1}\,K^{-1}}$]
- g acceleration of gravity [$\mathrm{m\,s^{-2}}$]
- H height of the porous layer [m]
- h average heat transfer coefficient for the bottom surface [$\mathrm{W\,m^{-2}\,K^{-1}}$]
- K permeability [$\mathrm{m^2}$]
- k effective thermal conductivity of the saturated porous medium [$\mathrm{W\,m^{-1}\,K^{-1}}$]
- L length of the heat sources [m]
- Nu overall Nusselt number, hH/k
- Pe Peclet number, U_0H/α
- Ra Rayleigh number, $Kg\beta(T_h - T_c)H/\alpha\nu$
- T temperature [k]
- t time [s]
- U dimensionless velocity in the x-direction, $u/U_0 = -\partial\psi/\partial Y$
- U_0 uniform velocity of the forced flow [$\mathrm{m\,s^{-1}}$]
- u velocity in the x-direction [$\mathrm{m\,s^{-1}}$]
- V dimensionless velocity in the y-direction, $v/U_0 = -\partial\psi/\partial X$
- v velocity in the y-direction [$\mathrm{m\,s^{-1}}$]
- X dimensionless distance on the x-axis, x/H
- Y dimensionless distance on the y-axis, y/H.

Greek symbols

- α effective thermal diffusivity of the saturated porous medium, $k/(\rho c)_f$ [$\mathrm{m^2\,s^{-1}}$]
- β coefficient of thermal expansion [$\mathrm{K^{-1}}$]
- ε porosity
- θ dimensionless temperature, $(T - T_c)/(T_h - T_c)$
- ρ fluid density [$\mathrm{kg\,m^{-3}}$]
- σ heat capacity ratio of the saturated porous medium to that of the fluid, $[\varepsilon(\rho c)_f + (1-\varepsilon)(\rho c)_s]/(\rho c)_f$
- τ dimensionless time, $t/(\sigma H^2/\alpha)$
- τ_p period of oscillation
- ψ stream function.

Subscripts

- c cooled surface
- f fluid phase
- h heated surface
- s solid phase.

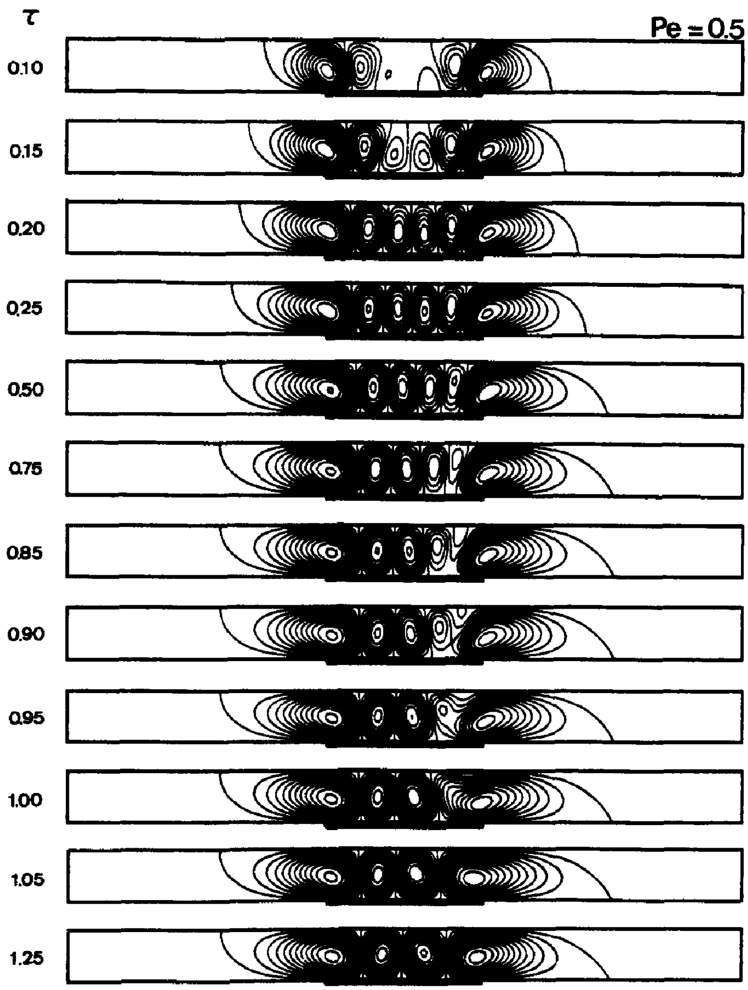

FIG. 2. Flow field for $A = 3$, $Ra = 100$ and $Pe = 0.5$ ($\Delta\psi = 1$).

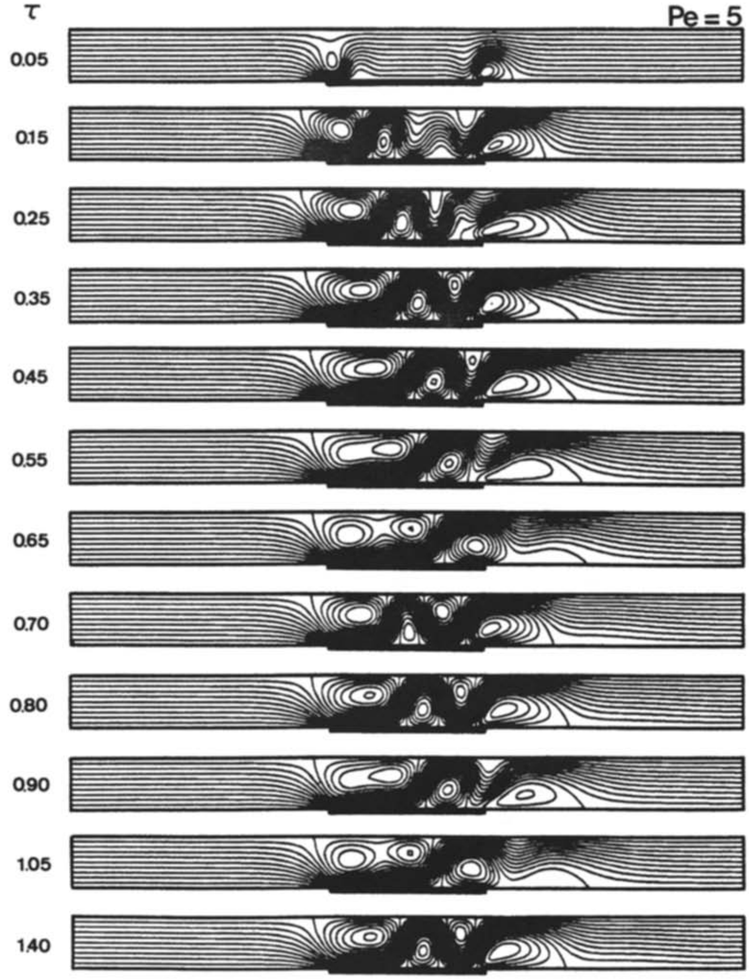

FIG. 3. Flow field for $A = 3$, $Ra = 100$ and $Pe = 5$ ($\Delta\psi = 0.1$).

respectively. At a small Peclet number, the flow and temperature fields are expected to retain the characteristics of natural convection. However, it is interesting to see that, initially, three pairs of recirculating cells are generated (Fig. 2). As reported for steady-state natural convection [5, 6], only two pairs of cells can exist in a stable condition, with each inner cell having an aspect ratio of 0.8. Therefore, it is suspected that the multicellular convection generated in this early stage is unstable. Indeed, as time proceeds, the additional pair of recirculating cells is swept away by the forced flow, leaving only two pairs of cells in the flow field.

As the Peclet number increases ($Pe = 5$), the strength of the forced flow becomes stronger, the third pair of cells is swept downstream long before their formation is completed (Fig. 3), and the remaining two pairs of cells are not stable. A periodic variation starts with the destruction of the second inner cell which is forced downstream by the primary flow, followed by the decomposition of the first cell and recombination of the inner cell and the downstream cell. At this point, the flow field is back to its original status and the process is repeated again with a period of about 0.5 dimensionless time units. It is evident that the oscillation of the flow field is primarily due to this periodic process of cell destruction and regeneration. It is interesting to point out that the periodic variation of the flow field reported here has also been observed in experiments [7].

For natural convection, Horne and O'Sullivan [8, 9] have reported flow instabilities in horizontal layers partially heated from below. They concluded that the disturbances result from a combination of cyclic triggering by predecessor cells which circulate around the domain and instability in the thermal boundary layer on the heated surface. However, in the present case, the disturbances once generated are forced downstream by the external flow, which greatly minimizes the triggering process of their circulation predecessors. Therefore, it can be concluded that the only mechanism for the oscillatory convection is the instability of the thermal boundary layer. While buoyancy effects tend to thicken the thermal boundary layer, the forced flow suppresses it. With the presence of the upper wall, this interaction is reinforced.

An overall Nusselt number can be defined based on the average heat transfer coefficient and is given by

$$Nu = \frac{hH}{k} = \int_0^A \left.\frac{\partial \theta}{\partial Y}\right|_{Y=0} \mathrm{d}X \tag{8}$$

which also represents the total heat flux over the heat source. The variations of the overall Nusselt number with time are presented in Fig. 4 as a function of the Peclet number. It is observed that the amplitude and period of the oscillation become smaller as the Peclet number increases. For $Pe \geqslant 18$, the flow and temperature fields stabilize again. It is at this

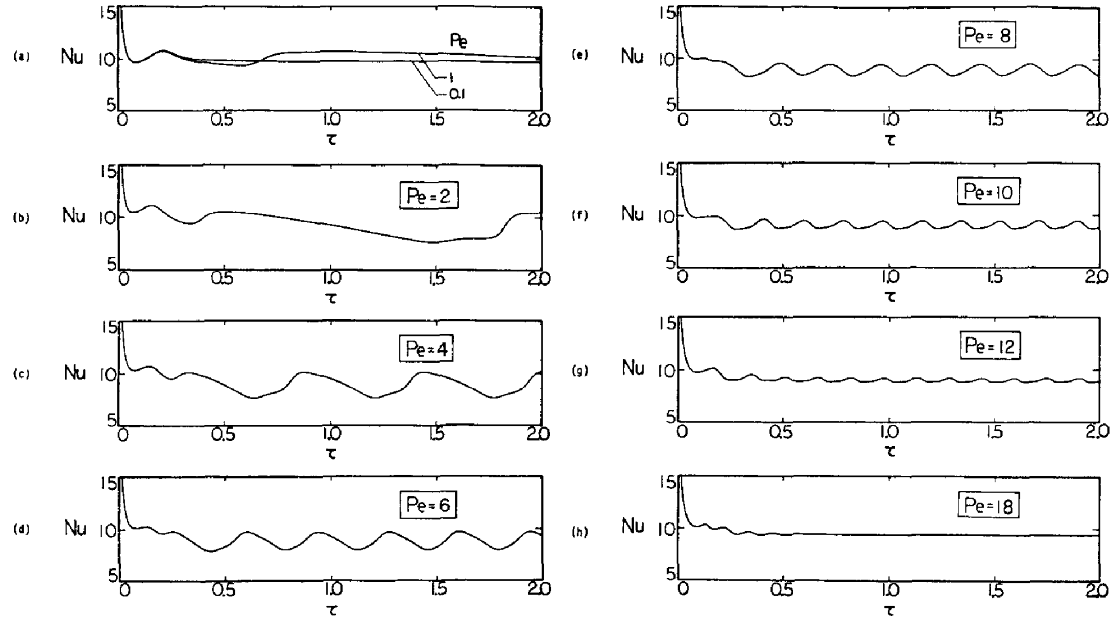

FIG. 4. Variation of overall Nusselt number with time, for $A = 3$.

point that the forced flow completely dominates the buoyancy effects. It is also interesting to note that for $Pe = 10$, the flow field is unstable. However, a steady-state solution has been reported in the previous study for this case [2]. It seems to contradict what we obtain here. A close examination of the temporal variations of the flow and temperature fields reveals that the solution we obtained earlier is a quasi-steady-state solution. Since the convergence criterion has been placed on the variations of the stream function and temperature, in some instances when the flow and temperature fields vary slowly, this condition can be met and the result is mistaken for the steady state. This can occur over some intervals when the changes in the variables are very small as shown in Fig. 4.

Upon correlating the numerical results, the period of oscillation as a function of Peclet number is

$$\tau_p = 2.41\,Pe^{-1.096} \tag{9}$$

while the amplitude is given by

In summary, numerical results have been presented for transient mixed convection over a horizontal porous layer with localized heating from below. For a small Peclet number ($Pe \leqslant 1$), the flow and temperature fields are stable, while for a larger Peclet number ($Pe > 1$), they exhibit an oscillatory variation due to the complicated interaction between the forced flow and the buoyancy effects. The period of the oscillation is found to be a function of the Peclet number. Flow and temperature fields stabilize again when the forced flow completely suppresses the buoyancy effects.

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